GEOMETRY.

PART I.

First three books of Plane Geometry.

(Answer Question 4, and any five others. But those who will take Part II and Part III should note the instructions given at the head of these divisions.)

1. Theorem: If two angles of a triangle are unequal, the sides opposite are unequal, and the greater side lies opposite the greater angle.

2. Theorem: If the sides of any polygon be produced so as to make an exterior angle at each vertex, the sum of these exterior angles is equal to four right angles.

3. Theorem: The medians of a triangle intersect at a common point, which lies two-thirds the way from each vertex to the middle point of the opposite side.

*4. Theorem: In the same circle, or in equal circles, two central angles are in the same ratio as their intercepted arcs. (Case II: When the arcs are incommensurable.)

Simply answer these four questions:

(a) Point out the equal variables.

(b) What are their limits?

(c) Why are their limits equal?

(d) What statement introduces, or creates, the variables?

5. Exercise: If a straight line be drawn through the point of

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*NOTE.—Since so very much of Plane and Solid Geometry depends on the Theorem of Limits, the chairman never gives an examination without one or more of its applications.

contact of two circles which are tangent to each other externally, terminating in their circumferences, the diameters drawn to its extremities are parallel. (Draw tangent at common point of contact.)

6. Theorem: Given two sides of a triangle, and the angle opposite to one of them, to construct the triangle.

A, the angle; m and n, the sides; n opposite A. Do not discuss; simply give six *constructions*.

Acute angle—five cases.

Right or obtuse angle—what single relation must m sustain to n, and why?

7. Exercise: To construct a triangle, having given the middle points of its sides.

8. Theorem: If any two chords be drawn through a fixed point within a circle, the product of the segments of one chord is equal to the product of the segments of the other.

PART II.

Last two books of Plane Geometry.

(Omit one question, but give Nos. 4 and 6 of Part I.)

1. Theorem: Two similar triangles are to each other as the squares of their homologous sides.

2. Exercise: In triangle, A B C, A B is equal to 25, B C is equal to 17, and C A is equal to 28. Find length of perpendicular from B to opposite side.

3. Theorem: A circle can be circumscribed about any regular polygon.

4. Corollaries

(a) The side of an inscribed equilateral triangle is equal to the radius of the circle multiplied by $\sqrt{3}$

(b) Having proved that the circumferences are to each other as their diameters, $\frac{C}{C'} = \frac{D}{D'}$, — prove $C = \pi D$ (π being a constant).

5. Exercise: The diagonals A C, B D, C E, D F, E A, and F B, of regular hexagon A B C D E F, form a regular hexagon whose area is equal to one-third the area A B C D E F.

Part III.

Solid Geometry.

(Omit two questions, but give No. 4 of Part I and No. 4 of Part II.)

1. Theorem: Every point in the bisecting plane of a diedral angle is equally distant from its faces.

(Give construction carefully.)

2. Theorem: The sum of the face angles of any convex polyedral angle is less than four right angles.

3. Exercise: Find the lateral edge, lateral area, and volume of a frustum of a regular triangular pyramid, the sides of whose bases are 18 and 6, respectively, and whose altitude is 24.

4. Theorem: The bases of a cylinder are equal.

5. Theorem: If one spherical triangle is the polar triangle of another, then the second spherical triangle is the polar triangle of the first.

6. Theorem: If the unit of measure for angles is the right angle, the area of a spherical triangle is equal to its spherical excess, multiplied by the area of a tri-rectangular triangle.

7. The lateral or total areas of two similar cones of revolution are to each other as the squares of their slant heights, or as the squares of their altitudes, or as the squares of the radii of their bases; and their volumes are to each other as the cubes of their slant heights, or as the cubes of their altitudes, or as the cubes of the radii of their bases.

(Consider only the total areas.)