

M A T H E M A T I C S.

ALGEBRA.

PART I—ALGEBRA TO QUADRATICS.

(Those who will take Part II may omit all but VI and VII.
Others may omit any two questions.)

$$\begin{array}{l}
 \text{I. } (+4)x(+3) = +12; (+4)x(-3) = -12; \\
 (-4)x(+3) = -12; (-4)x(-3) = +12.
 \end{array}
 \left. \vphantom{\begin{array}{l} (+4)x(+3) = +12; \\ (-4)x(+3) = -12; \end{array}} \right\} \begin{array}{l} \text{Give reasons for} \\ \text{signs in each} \\ \text{product.} \end{array}$$

II. Factor the following: $(a + b)^3 - (a - b)^2$;
 $a^3 + 25b^3 - 16c^3 - 9d^3 - 10ab - 24cd$;
 $(1 + a^3) + 2(1 - a)(1 + a)^2$.

III. Simplify the following:

$$\frac{1}{(x-y)(y-z)} - \frac{1}{(y-x)(x-z)} + \frac{1}{(z-x)(z-y)}$$

$$\text{IV. } \left\{ \begin{array}{l} \frac{x}{a} + \frac{y}{c} = 2b \\ \frac{x}{b} + \frac{z}{c} = 2a \\ \frac{y}{b} + \frac{z}{a} = 2c \end{array} \right. \quad \text{Find any two values}$$

V. A and B run a race of 336 yards. The first heat A gives B a start of 28 yards, and beats him by 2 seconds; the second heat, A gives B a start of 12 seconds, and is beaten by 48 yards. How many yards can each run in a second? (Two unknown quantities.)

VI. Prove $(a^m)^n = a^{m \cdot n}$ $\left\{ \begin{array}{l} 1. n, a \text{ positive integer.} \\ 2. n = \frac{p}{q}, p \text{ and } q \text{ being positive integers.} \\ 3. n = -s \text{ when } s \text{ is an integer or fraction.} \end{array} \right.$

VII. Simplify the following: $(2^{n+4} \times 2^x \times 2^n) \times (2^{-2} \times 2^{-n-2})$

VIII. Multiply the following: $6\sqrt{\frac{1}{3}} - 9\sqrt{\frac{1}{3}} + 10\sqrt{\frac{1}{3}}$ and $2\sqrt{\frac{1}{3}} + 3\sqrt{\frac{1}{3}} - 5\sqrt{\frac{1}{3}}$. Express result in simplest form.

XIX. Solve the following; $\sqrt{x + 2a\sqrt{4x + 3a^2}} = \sqrt{x} - 2a$.

PART II—ALGEBRA, BEYOND QUADRATICS.

(Omit any two, but give Nos. VI and VII of Part I.)

I. If $ax^2 + bx = c$, (a) Prove $x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$. (b) Solve the following equation, using principles of affected quadratics:

$$\sqrt[3]{3x - 2x^2} - \sqrt[3]{3x - 2x^2} = 2.$$

XII. A courier travels from P to Q in 12 hours. Another courier starts at the same time from a place 24 miles the other side of P, and arrives at Q at the same time as the first courier. The second courier finds that he takes half an hour less than the first to accomplish 12 miles. Find the distance from P to Q. (Simply determine equation.)

III. Solve the following equations: $\begin{cases} xy - (x - y) = 1. \\ x^2y^2 + (x - y)^2 = 13. \end{cases}$

IV. Find sum of Geometric Progression in terms of first term, No. of terms and rates,—i. e., Prove $s = \frac{ar^n - a}{r - 1}$

V. There are four numbers, the first three of which form an arithmetic progression, and the last three a geometric progression. The sum of the first and third is 2, and of the second and fourth 37. What are the numbers? (Let $x - y$, x , $x + y$ be first three.)

- VI. Expand the following to three terms (indicate fully, do not simplify): $\sqrt[3]{(x^{-\frac{3}{2}} - 3y^{\frac{1}{2}})^{-5}}$.
- VII. (a) Prove $\text{Log. } AB = \text{Log. } A + \text{Log. } B$.
- (b) In common system, prove that mantissa of a No. is not changed by multiplying or dividing the No. by any multiple of ten. Use: $10^{.4847} = 3.053$.